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Understanding the Inventory Cycle

by

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Abstract

Why is there inventory investment when its expected rate of return is strictly dominated by that of fixed-capital investment? Why is inventory investment procyclical at business-cycle frequencies but countercyclical at the very high frequencies (e.g., 2-3 quarters per cycle)? Why does the variance of production exceed the variance of sales at the business-cycle frequencies but not so at the high frequencies? Why is inventory investment so volatile relative to GDP at the high frequencies but not so at the business cycle frequencies? Explaining these seemingly paradoxical features of inventory behavior is of great importance because for many years economists have speculated that understanding inventory fluctuations may provide the key to understanding the business cycle. This paper provides a general equilibrium analysis on inventory cycles and their relations to the business cycle. I show that in an environment where production and fixed-capital investment cannot adjust instantaneously to respond to consumption demand shocks, firms opt to hold inventories in the short run so as to avoid stockout and to smooth production against demand uncertainty. These incentives for holding inventories in the short run have different effects on inventory cycles across different cyclical frequencies. At the high frequencies inventory fluctuations are dominated by the production-smoothing motive and at the business-cycle frequencies inventory fluctuations are dominated by the stockout-avoidance motive. Consequently, inventory investment appears to be countercyclical and volatile at the high frequencies but procyclical and relatively smooth at the business-cycle frequencies, production appears to be less volatile than sales at the high frequencies but more volatile than sales at the business-cycle frequencies. I also show that the inventory cycle and the business cycle are intimately related by sharing a common source of uncertainty - consumption demand, which leads to the phenomena that consumption Granger causes output and fixed investment, that consumption comoves with output and fixed investment, and that consumption appears to be smooth relative to output and fixed investment.

Keywords: Business cycles; Inventories; Demand shocks.

JEL classification: E13; E22; E32.

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1 Introduction

Why are there inventories? Unlike capital investment, inventory investment generates negative rate of return (due to depreciation and storage costs, for example), hence making it strictly dominated by capital investment in terms of prospective yield from national savings. Thus, from a resource allocation viewpoint, inventory investment is "inefficient". Despite the inefficiency, however, aggregate inventories are strictly positive, inventory investment is procyclical and it accounts for the bulk of fluctuations in GDP.

There are two prominent theories in the literature to explain the role of inventories: The production smoothing theory and the stockout-avoidance theory. According to the production smoothing theory, firms hold inventories in order to reduce production costs under demand uncertainty when costs of production are convex. So far, this theory has not been fared well empirically. There are two key facts contradicting this theory: Production appears to be more volatile than sales and inventory investment appears to be procyclical with respect to sales (e.g., see Blinder, 1986). If the motive for holding inventories is to smooth production against demand shocks, then variance of production is expected to exceed variance of sales and inventory investment is expected to be countercyclical with respect to sales. Inventory investment being procyclical with respect to sales suggests that firms over-produce systematically during booms when demand is high and over-cut output systematically during economic downturns when demand is low. Empirical facts thus appear to firmly reject the production-smoothing theory as a plausible explanation for the existence of inventories.[1]

According to the stockout-avoidance theory, firms hold inventories in order to avoid losses of opportunity for sales when production takes time and hence is incapable of responding to a demand shock instantaneously. With serially correlated demand shocks and sluggish adjustment in production, firms have incentive to produce to stock, resulting in excess volatility in production relative to sales and possible procyclical inventory investment, despite the inefficiency caused by depreciation and storage costs for holding inventories.

Careful analysis of quarterly aggregate data from the US and other OECD economies reveals, however, that inventory behavior exhibits far more complicated and puzzling features (to be documented shortly): 1) Inventory investment is procyclical only at relatively low cyclical frequencies such as the business-cycle frequencies (e.g., 8-40 quarters per cycle); it is countercyclical at very high frequencies (e.g., 2-3 quarters per cycle). 2) Production is more volatile than sales also

[1] But see Fair (1989) and Krane and Braun (1991) for some empirical evidence of production smoothing found in physical-product data of certain industries.

only around the business-cycle frequencies, but is less volatile than sales around the high frequencies. 3) Unlike capital investment or GDP, the bulk of the variance of inventory investment is concentrated around the high frequencies rather than around the business-cycle frequencies. These prominent features of aggregate inventory behavior indicate that inconsistence between the production-smoothing theory and data exists only in the relatively low frequency part of the data. The high frequency movements of the data appear to be fully consistent with the production smoothing theory.

The intriguing question is, however, does there exist a simple theory that can explain simultaneously the high-frequency and the low-frequency inventory cycles? A large body of the empirical literature on inventories is based on the linear-quadratic model, due to its goodness of fit (e.g., see Ramey and West 1997 for a recent review and application). A fundamental weakness of the mode is its lack of rigorous theory and *ad hoc* nature. On the other hand, the production-smoothing theory predicts countercyclical inventory investment and volatile sales relative to production across all cyclical frequencies, it alone cannot possibly provide explanations for inventory behavior at both the high frequencies and the business-cycle frequencies. It has proved quite challenging for providing models that can genuinely explain the apparent lack of production smoothing in the data. Important work includes introducing stockout-avoidance motives (e.g., Able 1985, Blanchard 1983, Kahn 1987, West 1986), supply-side shocks (e.g., Blinder 1986, Eichenbaum 1989), nonconvex costs of production (e.g., Ramey 1991). Among them, Kahn's work is perhaps most provocative. In a theoretical paper (Kahn 1987), Kahn claims that a stockout-avoidance motive together with serially correlated demand shocks are sufficient for explaining why variance of production can exceed variance of sales. In a follow-up paper, Kahn (1992) shows how this model can be supported empirically as a convincing explanation for the observed inventory behavior.

In spite of simplicity and elegance, Kahn's theory, however, is based on a partial equilibrium model with exogenous consumption demand and labor supply as well as highly restrictive assumptions regarding structural parameters of the economy, such as constant marginal costs of production, zero storage costs for holding inventories, and most importantly, there is no capital investment in his model. With exogenous labor supply, the tightness of the labor market plays no role in determining the variance of production. Due to exogenous demand, price is also exogenous and thus incapable of responding to inventory changes in the goods market, resulting in possible distortions of production and sales. With constant marginal costs of production, the motive for production smoothing is severely constrained *a priori*. These highly restrictive assumptions may have already begged the question as to why there is the lack of production smoothing, casting doubts on the robustness of Kahn's result. The assumption of no capital, in particular, seems to beg the very question as to why there exist inventories at the first place when resources could be allocated to more productive uses to yield higher expected rate of return. Therefore, although important insights are generated by Kahn's analysis, one cannot help wondering whether such a partial equilibrium model can be taken seriously as a rigorous model of inventory fluctuations and the business cycle.[2]

A few people have applied general equilibrium theory to study inventory fluctuations. Kydland and Prescott (1982) and Christiano (1987), for example, study inventory cycles in general equilibrium with consumption-leisure choice and capital accumulation. They show that general equilibrium business cycle models have the potential to explain the large volatility of inventory investment relative to GDP despite its disproportionately small steady-state share in aggregate output. Christiano (1987), in particular, shows that inventory investment can be both volatile and procyclical provided that the elasticity of substitution between capital and inventories is sufficiently low and that decisions for capital investment must be made based on imperfectly observed demand and supply shocks. Lingering doubts remain on the success of these general equilibrium analyses, however, for these models rely not only heavily on technology shocks to explain inventory fluctuations, but also exclusively on the assumption of inventories being a factor of production in order to explain the existence of inventories in equilibrium. As cautioned by Blinder (1986 and 1991), explanations for inventory and production volatilities based on supply shocks and explanations for the existence of inventories based on the production-factor argument come perilously close to assuming the conclusions.[3]

In this paper I provide a simple explanation, building on Kahn's (1987) insight, for the seemingly paradoxical features of inventory fluctuations observed at different cyclical frequencies. My explanation is based on general equilibrium theory in which aggregate savings can take either the form of inventory investment or the form of capital investment but with inventory investment being strictly dominated in expected rate of return by capital investment. Preference shocks is the only source of uncertainty in my model. The general equilibrium framework allows me to clarify issues that may be obscured or masked in Kahn's (1987) important analysis due to the highly

[2] Extensions of Kahn's (1987) model to allowing for storage costs and more general demand shock process are worked out by Maccini and Zabel (1996). These extensions, however, still rely on partial equilibrium analysis where demand is exogenous and capital investment plays no role in firm's resource allocation decisions.

[3] In these general equilibrium models, as pointed out by Christiano's (1987), inventories also exist for another reason, which is for being a buffer against economic shocks when production and fixed-capital investment are not capable of responding fully to shocks instantaneously due to information frictions. This is similar to the stockout-avoidance function studied by Kahn (1987). But such a function of inventories is not distinguished from being a factor of production in these models, hence this role of inventories is not fully explored by these authors.

restrictive nature of assumptions adopted there for analytical tractability. I completely strip away the role of technology shocks and the role of inventories as a factor of production from the models of Kydland and Prescott (1982) and Christiano (1987), but retain their information structure conventional to the literature: decisions about production and capital investment must be made in advance before demand uncertainty is resolved. This information structure allows inventories to be valued in equilibrium under the rate-of-return dominance and the stockout-avoidance motive to be relevant under a non-negative constraint on inventories. In addition, a standard utility function with consumption-leisure choice and a standard neoclassical production function with capital and labor are assumed to encompass the production-smoothing motive.

I show that Kahn's (1987) insight continues to hold in general equilibrium as long as certain conditions are met so that motives for avoiding stockouts can dominate motives for smoothing production in the longer run (i.e., at business-cycle frequencies). These conditions imply that in equilibrium it is not optimal to plan to hold too many inventories in advance before demand uncertainty is resolved (i.e., planned production closely follows expected sales). To be more specific, Kahn's (1987) conclusion that a stockout-avoidance motive and serially correlated demand shocks are sufficient for the variance of production to exceed the variance of sales can hold in general equilibrium only if the following conditions are met: 1) Either the marginal cost of production is constant and storage costs for holding inventories are strictly positive; 2) Or costs of production are convex but the marginal cost for holding inventories is sufficiently large; 3) Or there exists any other asset that dominates inventory investment in expected rate of return so that firms have no incentive to plan for holding inventories in the long run when decisions for production are made.

Among these three sets of conditions, the last one is the most interesting and realistic. The first two sets of conditions are less likely to be always satisfied in reality, but they amount to the same thing: Namely, if the marginal cost of production is constant or the marginal cost of holding inventories is large, then firms will have less incentive to use inventories to smooth production against expected demand shocks, hence they opt to cut back production when expected demand is low so that planned supply moves closely with expected demand. A consequence of this is that production is more volatile than sales. If production costs are convex or marginal storage costs for holding inventories are small, however, firms will find it optimal to use inventories to smooth production. A consequence of this is that production is less volatile than sales. However, if there exists any other asset (such as capital) that dominates inventories in expected rate of return in the long run so that inventories are valued *only* in the short run, then it is optimal not to plan to hold inventories too early in advance in order to smooth production, weakening the production-smoothing motive at the business-cycle frequencies.[4] A consequence of this is that inventory investment may become procyclical and the variance of production may exceed the variance of sales in spite of convex production costs and small inventory-holding costs.

Thus, in an environment with capital investment that dominates inventory investment in expected rate of return, I show that serially correlated demand shocks can explain inventory fluctuations at both the high frequency and the low frequency intervals despite convex production costs and small inventory holding costs. At the high frequencies, due to the sluggish adjustment in both production and capital investment in the short run, inventories serve as the only buffer against consumption shocks, hence resulting in volatile and countercyclical changes in inventories and less volatile production relative to sales. At the business cycle frequencies, however, since the production-smoothing motive for holding inventories is diminished by capital investment (which tends to comove with consumption under serially correlated preference shocks), the model behaves as if there were only stockout-avoidance motives as in Kahn's (1987) model.

The information structure that decisions about production and capital investment must be made in advance before consumption-demand uncertainty is resolved is pivotal for the success of the general equilibrium model in explaining the existence and behavior of inventories. Although this information structure is quite intuitive and conventional to the literature, it nonetheless warrants a serious test. A striking implication of the information structure is that consumption Granger-causes both production and capital investment but not vise versa. Such an implication is tested using quarterly postwar US aggregate data. The data strongly support this information structure. Hence, the very condition for explaining the existence and characteristics of inventories in general equilibrium and the phenomenon that consumption appears to cause the business cycle are shown to be intrinsically related.

An important remaining question is, can many of the defining features of the business cycle emphasized by the existing RBC literature also be explained by consumption shocks alone? This question is important because the relationship between inventory fluctuations and the business cycle may hinge crucially on whether they share a common source of disturbances. Preliminary analysis shows that the answer to the question is "yes". To demonstrate this, I show in the paper that the same general equilibrium business cycle model driven solely by preference shocks can qualitatively explain many salient features of the business cycle emphasized by the RBC literature, such as the observed positive comovements among output, consumption, employment and

[4]Production can be smoothed via capital investment in the long run. However, since capital investment moves together with consumption under serially correlated preference shocks (to be shown shortly), it forces production to be volatile rather than smooth.

capital investment during recessions and booms, and the stylized fact that consumption appears to be smooth relative to income and capital investment. The logic behind this has already been provided by Wen (2002). The intuition is that when consumption shocks are serially correlated, higher current consumption demand implies also higher future consumption demand which requires higher future output. Then resources allocated to national savings (capital investment) may increase (rather than being crowded out by current consumption demand) so that future production capacity can increase. Hence, a positive change in consumption demand can induce a positive increase in investment demand, which reinforces the initial increase in consumption demand and results in a multiplier effect on output and employment, generating not only more variable production than sales but also positive comovements for consumption, production and capital investment. Furthermore, because marginal utilities of future consumption may outweigh that of current consumption when shocks are highly serially correlated, savings (investment) may increase more than current consumption does with respect to changes in national income, resulting in variance of consumption being smaller than variances of output and capital investment. Hence, serially correlated demand shocks alone can explain, at least qualitatively, both the inventory cycle and the business cycle with respect to their most prominent features.

The rest of the paper is organized as follows. Section 2 documents the dramatically different nature of inventory fluctuations at different cyclical frequencies in the US and other OECD countries. Section 3 embeds Kahn's (1987) stockout-avoidance theory into a prototype general equilibrium model without capital investment to help interpret these empirical facts. This model helps to understand the insights as well as the limitations of Kahn's (1987) partial-equilibrium stockout-avoidance model. Section 4 extends the basic model to allow for capital investment and uses it to explain inventory fluctuations. Section 5 discusses possible links between inventory fluctuations and the business cycle and shows that consumption demand shocks alone can qualitatively explain many prominent features of the business cycle. Section 6 concludes the paper with remarks on further research.

2 Reality: Inventory Cycles at High and Low Frequencies

This section documents the stark differences in inventory dynamics across high and low frequencies for post-war US and some OECD economies.[5] I use the band-pass filter (Baxter and

[5] All data are seasonally adjusted, quarterly real data. Total sales is defined as total output minus inventory investment. The US data are taken from Citibase (1960:1 - 1994:4). The rest are taken from OECD data bank (1960:1 - 1994:4). Countries with missing data or very short data series (less than 30 observations) are excluded from the sample.

King, 1995) to isolate movements of aggregate inventory investment, production, and sales in the frequency range of 2-3 quarters per cycle (called the "high frequency" interval) and the frequency range of 8-40 quarters per cycle (called the "business-cycle frequency" interval).[6] Before applying the filter, all data series are transformed in a way so that their definitions match the definitions adopted in the theoretical model of the paper. In particular, all data series except inventory investment are logged and detrended using a linear time trend. Since inventory investment series contain negative values and do not have noticeable time trend, it is normalized (divided) by the median of output series in the respective country (region). Table 1 reports the relative volatilities of production with respect to sales and correlations between inventory investment and sales.[7] Several striking patterns of inventory and production behavior emerge from the table:

1) At the high frequencies (1st column) production is less volatile than total sales. The ratio of standard deviations between output and sales, for example, is 0.91 for OECD and 0.83 for European countries as a whole. The only exceptions at the individual country level are the United States and Finland.

2) At the high frequencies (2nd column) inventory investment is strongly countercyclical with respect to sales. The correlation between inventory investment and sales, for example, is -0.43 for OECD countries and -0.51 for European economies. This is true for all individual countries listed.

3) In stark contrast, at the business cycle frequencies (3rd column) production is more volatile than sales. The ratio of standard deviations between GDP and sales, for example, is 1.39 for OECD and 1.55 for European countries as a whole. There is no exception for any individual countries in the sample.

4) At the business cycle frequencies (4th column), inventory investment appears to be positively correlated with sales. Except for Austria and Switzerland, the correlations between inventory investment and sales are all positive. It is, for example, 0.58 for OECD and 0.47 for European countries as a whole.

These seemingly paradoxical patterns of inventory behavior and the associated behavior of output-sales volatility ratios demand serious explanations. It has been a consensus in the existing

[6] The results do not change dramatically when the high-frequency interval is extended from 2-3 quarters per cycle to 2-4 quarters per cycle and the business-cycle frequency interval is extended from 8-40 quarters per cycle to 6-100 quarters per cycle. The band-pass filter uses a maximum lag length of $k = 8$ as the truncation window parameter, implying 8 observations are lost at each end of the data series. This choice is based on the length of samples. See Baxter and King (1995) for discussions on this issue.

[7] These statistics do not change significantly if the band-pass filter is applied directly to the raw data series without the aforementioned transformation.

literature that inventory investment is procyclical and production is more volatile than sales (e.g., see Blinder 1986 and 1991, and Ramey and West 1997). But careful re-examination of aggregate data showed that these two stylized characteristics of inventory behavior hold true only at the lower cyclical frequencies (frequencies lower than 6 or 8 quarters per cycle). The opposite, however, is true at the very high frequencies (frequencies higher than 4 or 3 quarters per cycle). It needs to be emphasized that these high-frequency movements are not seasonal movements, as the data used in producing table 1 are all seasonally adjusted.

These features of production and inventory behavior identified at the high frequencies are fully consistent with the conventional production smoothing theory, while those identified at the business-cycle frequencies seem fully consistent with the stockout-avoidance theory. The intriguing question is, however, does there exist a single theory that can explain simultaneously these inventory cycles across different frequencies? Or can such a theory, if exists, also provide keys for understanding the business cycle in general? These issues are addressed in the following sections.

3 Theory: A Basic Model without Capital

This section embeds Kahn's (1987) stockout-avoidance inventory theory into a simple general equilibrium model without fixed capital. The purpose of this section is two folded: To develop insights on resolving the inventory puzzles by dissecting the roles inventories may play both in preventing stockouts and in smoothing production; Second, to provide a dynamic programming technique for solving general equilibrium inventory models with non-negativity constraints. This method will be used intensively in the next section to solve for a more general model with both inventory investment and capital investment. In what follows, I first state the problem in a decentralized economy. I then show how to map the model into a representative-agent framework in which dynamic programing is easier to carry out.

A. Firm's Program

A representative firm chooses production Y and the amount of sales Z to maximize expected profit. The production technology is given by

$$Y_t = AN_t^{\alpha}, \qquad 0 < \alpha \leq 1;$$

where N is employment. The firm takes goods price (P_t), wage rate (W_t), as well as demand C_t as given. For the stockout-avoidance motive to be relevant, it is assumed (as in Kahn, 1987) that employment decisions are made one period in advance, hence period-t output cannot respond to demand shocks in period t. Period t profit is given by

$$Q_t = P_t Z_t - W_t N_t - P_t \frac{\phi}{2} S_t^2, \tag{3.1}$$

where $S_t \geq 0$ is inventory holdings determined in period t and the quadratic term, $\frac{\phi}{2} S_t^2$, reflects increasing marginal storage costs (measured in output) for carrying inventories. The size of marginal storage cost is controlled by the parameter $\phi \geq 0$.[8] The total amount of planned stock available to meet demand in period t is the sum of production and non-depreciated inventories carried from last period: $Y_t + (1-\delta)S_{t-1}$, where $\delta \in [0, 1]$ is the depreciation rate. In case actual demand is less than planned stock for sale, there is inventory accumulation, $S_t > 0$. Otherwise there is stockout and $S_t = 0$. Hence we have

$$Z_t = \min \{C_t, [Y_t + (1-\delta)S_{t-1}]\}. \tag{3.2}$$

The law of motion for the inventory stock is given by

$$S_t = Y_t + (1-\delta) S_{t-1} - Z_t. \tag{3.3}$$

The optimal program for the firm is then to solve

$$\max_{\{N_t\}} E_{t-1} \left\{ \max_{\{Z_t, S_t\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[P_t \left(Z_t - \frac{\phi}{2} S_t^2 \right) - W_t N_t \right] \right\} \right\}$$

subject to (3.2) and (3.3). When $\alpha = 1, \delta = 0$ and $\phi = 0$, this model becomes identical to the model studied by Kahn (1987).

To facilitate embedding this model into a general equilibrium framework, I change it by replacing the constraint (3.2) by a non-negativity constraint on inventories:

$$S_t \geq 0. \tag{3.4}$$

Then use equation (3.3) to substitute out Z_t in the objective function of the firm, the new program for the firm is to solve

$$\max_{\{N_t\}} E_{t-1} \left\{ \max_{\{S_t\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[P_t \left(A N_t^{\alpha} + (1-\delta) S_{t-1} - S_t - \frac{\phi}{2} S_t^2 \right) - W_t N_t \right] \right\} \right\}$$

subject to the non-negativity constraint (3.4). To complete the model, a representative consumer's program is stated below. I will show later how to derive Kahn's (1987) result as a special case from the general equilibrium model.

[8] The inventory holding cost is assumed to be quadratic so as to distinguish it from depreciation costs of inventories.

B. Consumer's Program

Taking as given the market prices $\{P_t, W_t\}$ and the profit income received from the firm (Q_t), a representative consumer chooses consumption demand (C_t) and labor supply (L_t) to solve

$$\max_{\{L_t\}} E_{t-1} \left\{ \max_{\{C_t\}} \left\{ \Theta_t \log C_t - a \frac{L_t^{1+\gamma}}{1+\gamma} \right\} \right\}$$

subject to the budget constraint

$$P_t C_t = W_t L_t + Q_t,$$

where the profit income is given by

$$Q = P_t \left(A N_t^{\alpha} + (1-\delta) S_{t-1} - S_t - \frac{\phi}{2} S_t^2 \right) - W_t N_t,$$

and where Θ_t is a random shock to the marginal utility of consumption. It is assumed that the consumption-demand shock follows a log-stationary AR(1) process:

$$\log \Theta_t = \rho \log \Theta_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \left(0, \sigma^2\right).$$

Since there exists no asset other than inventories, I have made the assumption that the consumer does not make independent saving decisions and that the firm redistributes inventories to the consumer as part of profit income.[9]

C. Competitive Equilibrium as Solution to Planner's Problem

A competitive equilibrium is a set of price sequences $\{W_t, P_t\}_{t=0}^{\infty}$ and quantity sequences $\{C_t, N_t, L_t, S_t, Z_t\}_{t=0}^{\infty}$ that solves both the firm's program and the consumer's program, and satis-

[9] An alternative approach would be to let the consumer choose inventories as a saving device subject to the storage costs and a non-negativity constraint on inventories. Namely, the consumer (taking market prices as given) solves:

$$\max_{\{L_t\}} E_{t-1} \left\{ \max_{\{C_t, M_t\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\Theta_t \log C_t - a \frac{L_t^{1+\gamma}}{1+\gamma} \right] \right\} \right\}$$

subject to

$$C_t + M_t = \frac{W_t}{P_t} L_t + (1-\delta) M_{t-1} - \frac{\phi}{2} M_t^2 + Q_t'$$

and

$$M_t \geq 0;$$

where the real profit income is given by

$$Q_t' = Y_t - \frac{W_t}{P_t} N_t,$$

and

$$M_t = S_t$$

in equilibrium.

fies both the labor-market clearing condition,

$$L_t = N_t,$$

and the goods market clearing condition,

$$C_t + S_t = Y_t + (1-\delta)\, S_{t-1} - \frac{\phi}{2} S_t^2,$$

as well as the non-negative constraint (3.4). Without market distortions, the first welfare theorem implies that the competitive equilibrium can be computed as a solution to a planner's problem in which a planner chooses consumption (C_t), labor hours (N_t) and inventory investment (S_t) to solve:

$$\max_{\{N_t\}_{t=0}^{\infty}} E_{t-1} \left\{ \max_{\{C_t, S_t\}_{t=0}^{\infty}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \Theta_t \log C_t - a \frac{N_t^{1+\gamma}}{1+\gamma} \right\} \right\} \right\}$$

subject to

$$C_t + S_t = A N_t^{\alpha} + (1-\delta)\, S_{t-1} - \frac{\phi}{2} S_t^2, \tag{3.5}$$

and

$$S_t \geq 0. \tag{3.6}$$

Since inventory investment yields negative rate of return due to depreciation and storage costs, the non-negativity constraint (3.6) is highly relevant (since the planner has incentive to borrow from future income, it will result in negative savings without the non-negativity constraint, especially in case of positive demand shocks).[10] The first-order conditions are given by (where Λ_t and Γ_t denote Lagrangian multipliers associated with (3.5) and (3.6) respectively):

$$\frac{\Theta_t}{C_t} = \Lambda_t \tag{3.7}$$

$$E_{t-1}\left(a N_t^{\gamma}\right) = E_{t-1}\left(\Lambda_t \alpha A N_t^{\alpha-1}\right) \tag{3.8}$$

$$\Lambda_t \left(1 + \phi S_t\right) = \beta(1-\delta) E_t \Lambda_{t+1} + \Gamma_t \tag{3.9}$$

$$C_t + S_t - (1-\delta) S_{t-1} = A N_t^{\alpha} - \frac{\phi}{2} S_{t-1}^2 \tag{3.10}$$

$$\begin{aligned} S_t = 0 \quad & \text{if and only if } \Gamma_t > 0 \\ S_t \geq 0 \quad & \text{if and only if } \Gamma_t = 0 \end{aligned} \tag{3.11}$$

plus a transversality condition,

$$\lim_{T \to 0} \beta^T \left[\Lambda_T \left(1 + \phi S_T\right)\right] S_T = 0. \tag{3.12}$$

[10]In this paper I use preference shocks and demand shocks interchangeably.

Note that in the steady state optimal inventory holdings (S^*) equal zero since

$$\Gamma^* = \Lambda^* \left(1 + \phi S^* - (1 - \delta)\beta\right) > 0.$$

This implies that inventories are valued by the planner only in the short run (off the steady state) as long as uncertainty exists to prevent planned supply from being equal to actual demand. Inventories are valued in the short run despite a negative rate of return on inventory investment because inventories can be used to avoid stockouts and to smooth production against demand uncertainty. With perfect foresight (in the steady state), however, the optimal inventory stock is zero since planned production will always be able to match actual demand in the absence of uncertainty.

The interpretations of the first-order conditions are straightforward. In equation (3.7) the marginal utility of consumption equals the shadow price (Λ) of goods. In equation (3.8) the marginal cost of leisure equals the marginal product of labor in utility terms in expectation based on information available in period $t-1$. In equation (3.9) the marginal cost of keeping one extra unit of inventories is $\Lambda_t (1 + \phi S_t)$ due to the lost opportunity for consumption in period t (Λ_t) and storage costs (ϕ), but the marginal benefit for having $(1-\delta)$ extra units of inventories available for consumption in the next period depends on the goods price (marginal utility of consumption) expected to prevail in the next period, $E_t\Lambda_{t+1}(1-\delta)$. However, the marginal cost for keeping one extra unit of inventories in period t must be adjusted by a slackness multiplier Γ_t: it is positive when stockout occurs and zero when stockout does not occur. In equilibrium, the marginal cost of keeping inventories equals to the expected marginal benefit.

D. Solution Method

Since analytical solutions are not available, I use the standard method in the RBC literature to compute equilibrium decision rules near the steady state. In particular, I use the method of log linearization (King, Plosser, and Rebelo, 1988) to study the local dynamics of the model. In the linearized model, variables such as X_t are expressed in terms of deviations around the log of their steady-state values. Using lower-case letters to denote such variables, then $x_t \equiv \log X_t - \log X^*$. Since the inventory stock S_t has a steady-state value of zero, it cannot be linearized around the log of its steady state value. For technical reasons, it is linearized around zero instead. For example, in the intertemporal Euler equation (3.9), the non-negativity constraint on inventories creates a wedge ($\Gamma_t \geq 0$) between $\Lambda_t (1 + \phi S_t)$ and $\beta (1-\delta) E_t\Lambda_{t+1}$:

$$\begin{aligned} \Lambda_t (1 + \phi S_t) &= \beta (1-\delta) E_t\Lambda_{t+1} + \Gamma_t \\ &\leq \beta (1-\delta) E_t\Lambda_{t+1}. \end{aligned}$$

Taking first-order Taylor expansion around $\log \Lambda_t = \log \Lambda^*$ and $S_t = 0$ gives

$$\lambda_t + \phi s_t \leq \beta (1-\delta) E_t \lambda_{t+1},$$

where s_t is proportional to S_t near the steady state, so the non-negatively constraint on inventories in the original nonlinear economy ($S_t \geq 0$) can be replaced by $s_t \geq 0$ in the linear economy. Define

$$\begin{aligned} \pi_t &\equiv \beta (1-\delta) E_t \lambda_{t+1} - (\lambda_t - \phi s_t) \\ &\geq 0, \end{aligned}$$

then the original slackness condition on inventories can be redefined as $\pi_t s_t = 0$. The rest (such as the resource constraint) can be linearized in a similar fashion. With these modifications, the set of linearized first-order conditions are given by

$$\theta_t - c_t = \lambda_t \tag{3.13}$$

$$(1+\gamma-\alpha) E_{t-1} n_t = E_{t-1} \lambda_t \tag{3.14}$$

$$\lambda_t + \phi s_t = \beta(1-\delta) E_t \lambda_{t+1} + \pi_t \tag{3.15}$$

$$\frac{1}{y^*} s_t - \frac{1}{y^*} (1-\delta) s_{t-1} = \alpha n_t - c_t \tag{3.16}$$

$$\begin{aligned} s_t = 0 &\quad \text{if and only if } \pi_t > 0 \\ s_t \geq 0 &\quad \text{if and only if } \pi_t = 0 \end{aligned} \tag{3.17}$$

$$\theta_t = \rho \theta_{t-1} + \varepsilon_t;$$

where y^* ($\equiv Y^*$) is the steady-state value of output. In the linearized system, a variable (x_t) taking a positive (negative) value implies that its upper-case counterpart (X_t) is above (below) its steady-state value whereas a value of zero simply means that its upper-case counterpart is constant (equal to its steady state value). The variable s_t can also be interpreted in a similar way. For example, $s_t = 0$ implies that the inventory stock (S_t) remains constant with respect to its steady state value (which is zero) and $s_t > 0$ implies that S_t is above zero (positive).[11] Notice that the linearization procedure results in variables that are consistent with the definitions of the data series documented previously in table 1, hence empirical comparisons between the model and the data can be carried out straightforwardly. To simplify the analysis without loss of insight, I arbitrarily set $y^* = 1$ in this section. This parameter will be calibrated in numerical studies in

[11] The interpretation of π_t is however a bit different from $(\log \Pi_t - \log \Pi^*)$ due to the redefinition of the slackness condition. Without loss of accuracy, however, π_t can be treated as a completely different slackness multiplier viable only for the inventory stock (s_t) in the log-linearized system.

other sections. Equilibrium decision rules of the model are derived according to the algorithm (general steps) provided in Appendix 0.

The following two subsections discuss the implications of the simple inventory model for inventory cycles and compare them with the results obtained by Kahn (1987). Since optimal decision rules take very simple forms when $\phi = 0$, which makes interpretation easier, I discuss the case with zero storage costs first. Implications for positive storage costs are discussed next.

E. Results with Zero Storage Costs

Proposition 1 *With zero storage costs, equilibrium decision rules for labor, production, consumption (sales), and the inventory stock are given by:*

$$n_t = \max\left\{0, \frac{1}{1+\gamma}\left[\rho\theta_{t-1} - (1-\delta)\, s_{t-1}\right]\right\}$$

$$y_t = \max\left\{0, \frac{\alpha}{1+\gamma}\left[\rho\theta_{t-1} - (1-\delta)\, s_{t-1}\right]\right\},$$

$$c_t = \min\left\{\theta_t, \max\left\{(1-\delta)\, s_{t-1}, (1-\delta)\, s_{t-1} + \frac{\alpha}{1+\gamma}\left[\rho\theta_{t-1} - (1-\delta)\, s_{t-1}\right]\right\}\right\}$$

$$s_t = \max\left\{0, \max\left\{(1-\delta)\, s_{t-1} - \theta_t, (1-\delta)\, s_{t-1} - \theta_t + \frac{\alpha}{1+\gamma}\left[\rho\theta_{t-1} - (1-\delta)\, s_{t-1}\right]\right\}\right\}$$

Proof. See Appendix 1.■

It is clear that the variance of production can never exceed the variance of sales in this model if α is small enough (namely, if the production function is sufficiently concave) or if γ is large enough (namely, if labor supply is sufficiently inelastic), because $\lim_{\alpha\to 0} y_t = \lim_{\gamma\to\infty} y_t = 0$ and $\lim_{\alpha\to 0} c_t = \lim_{\gamma\to\infty} c_t = \min\left\{\theta_t, (1-\delta)\, s_{t-1}\right\}$. This implies that tightness in the labor market and the shape of the production technology matter in determining the relative volatilities of production and sales. This is not obvious in Kahn's (1987) partial equilibrium model.

The decision rules reflect both a stockout-avoidance motive and a production-smoothing motive. Too see this, notice that the decision rule for employment can be rewritten as:

$$n_t = \begin{cases} \frac{1}{1+\gamma}\left[E_{t-1}\theta_t - (1-\delta)\, s_{t-1}\right], & \text{if } E_{t-1}\theta_t > (1-\delta)\, s_{t-1}; \\ 0, & \text{if } E_{t-1}\theta_t \leq (1-\delta)\, s_{t-1}. \end{cases}$$

Namely, if expected future demand is above the existing stock ($E_{t-1}\theta_t > (1-\delta)\, s_{t-1}$), it is optimal to increase current production (employment) so as to avoid stockout (hence $n_t > 0$).[12] On the other hand, if expected future demand is below the existing stock ($E_{t-1}\theta_t \leq (1-\delta)\, s_{t-1}$), then

[12]The size of the increase in employment, however, depends on the tightness of the labor market. When γ is large (labor supply is less elastic), employment (production) is less responsive to a change in expected future demand.

it is optimal to keep production constant when there are no storage costs for holding inventories (hence $n_t = 0$).[13] Accordingly, the optimal inventory policy at $t-1$ is to plan to hold inventories ($E_{t-1}s_t \geq 0$) so as to facilitate smoothing production. The employment decision rule thus suggests that a stockout-avoidance motive is necessary but may not be sufficient to give rise to more volatile production than sales.

To gain further insight in why a stockout-avoidance motive may not be sufficient for production counter-smoothing, consider the special case where $\alpha = 1$ and $\gamma = 0$ so that motives for production smoothing in the model are reduced to lowest possible limit. Note that this is the case coincide with Kahn's (1987) parameterization. Without loss of generality, I also assume $\delta = 0$. I will show that even in such a case the variance of sales may still exceed the variance of production, contrary to Kahn's conclusion, due to the lack of storage costs for holding inventories. The reason is that the lack of inventory holding costs encourages firms to use inventories to smooth production. With these parameter values, the equilibrium decision rules become:

$$y_t = \max\{0, \rho\theta_{t-1} - s_{t-1}\},$$

$$c_t = \min\{\theta_t, \max\{\rho\theta_{t-1}, s_{t-1}\}\},$$

$$s_t = \max\{0, \max\{-\varepsilon_t, s_{t-1} - \theta_t\}\}.$$

The threshold level of expected demand determining the actions is $E_{t-1}\theta_t$ $(= \rho\theta_{t-1})$, which in turn is determined by the existing stock level, s_{t-1}.

To see this, consider two possible cases. A) The existing inventory stock is high relative to the expected demand (i.e., $s_{t-1} \geq \rho\theta_{t-1}$), then the decision rules are:

$$\begin{aligned} y_t &= 0, \\ c_t &= \min\{\theta_t, s_{t-1}\}, \\ s_t &= \max\{0, s_{t-1} - \theta_t\}; \end{aligned} \tag{A}$$

meaning that the planner opts to smooth production completely ($y_t = 0$) rather than to decrease production despite the risk of a stockout is low tomorrow (given s_{t-1} is relatively large), suggesting that production does not closely follow expected sales. Notice that the threshold is exactly the

[13]Since the lower-case variables denote deviations from the steady state, hence zero values in n_t and y_t simply mean that employment and production in the original upper-case economy are constant. Namely, $n_t = 0$ implies $N_t = N^*$ (> 0), and $y_t = 0$ implies $Y_t = A(N^*)^\alpha$.

As will be shown shortly, however, production will decrease in response to a lower expected demand if there exists storage costs.

expected sales, since

$$E_{t-1}c_t = \min\{E_{t-1}\theta_t, \max\{\rho\theta_{t-1}, s_{t-1}\}\} = \min\{\rho\theta_{t-1}, s_{t-1}\} = \rho\theta_{t-1}.$$

Obviously, the variance of production is zero, less than the variance of sales.

B) The existing inventory stock is low relative to expected demand (i.e., $s_{t-1} < E_{t-1}\theta_t$), then the decision rules are:

$$\begin{aligned} y_t &= \rho\theta_{t-1} - s_{t-1}, \\ c_t &= \rho\theta_{t-1} + \min\{0, \varepsilon_t\}, \\ s_t &= -\min\{0, \varepsilon_t\}; \end{aligned} \tag{B}$$

meaning that it is optimal to produce ($y_t = E_{t-1}\theta_t - s_{t-1} > 0$), so as to avoid possible stockouts in period t. Notice that in this case production moves closely with (or targets) the expected demand so that planned supply, $y_t + s_{t-1}$, equals expected sales, $E_{t-1}c_t$:

$$E_{t-1}c_t = \rho\theta_{t-1} + \min\{0, E_{t-1}\varepsilon_t\} = \rho\theta_{t-1} = y_t + s_{t-1}.$$

In this case the variance of production exceeds the variance of sales as long as $\rho > 0$ (see below for a proof).

In addition, the general decision rule for inventories,

$$s_t = \max\{0, \max\{-\varepsilon_t, s_{t-1} - \theta_t\}\},$$

shows that inventory stock is negatively correlated with demand shocks, suggesting the buffer-stock role of inventories. Since inventories in this general equilibrium model also serve as a saving device to transfer resources from the current to the future so as to defer consumption (otherwise current consumption has to absorb the inventory stock when the marginal utility of consumption is low), they play a triple role in this economy: to function as a saving device to engage in intertemporal maximization of expected utilities, to avoid stockout, and to smooth production.

Clearly, production will be more volatile than sales in this model only if stockout-avoidance motives dominate production-smoothing motives so that production moves closely with expected sales as in case B). When planned inventory holding is positive (i.e., when production does not move closely with expected falls in sales), the motive to smooth production dominates the motive to avoid a stockout, hence the variance of production may be less than the variance of sales (as in case A). Consequently, in order to ensure that the variance of production exceeds the variance of sales in this model, some type of economic forces need to operate to suppress the production-smoothing motives (especially in the case when expected demand is low) so that firms opt to keep production in line with expected demand by reducing production rather than to accumulate inventories by keeping production constant.

The analysis indicates that Kahn's (1987) conclusion that a stockout-avoidance motive and serially correlated demand shocks are sufficient for the variance of production to exceed the variance of sales is based on an implicit precondition: that planned supply ($y_t + s_{t-1}$) must equal expected demand ($E_{t-1}c_t$) in every period t so that $E_{t-1+j}s_{t+j} = 0$ for all j. This conditions amounts to insure that firms can decrease production sufficiently when expected demand is low so that no inventories are expected to be held in advance (i.e., $E_{t-1}s_t = 0$). But this condition does not hold here with zero storage costs unless demand shocks are always expected to be positive (i.e., $E_{t-1+j}\theta_{t+j} > s_{t-1+j}$ for all j) so that stockouts are expected to occur in every period (*ex anti*). In such a case the decision rules take only the form in (B). Then, since $E_{t-1+j}\theta_{t+j} > s_{t-1+j}$ for all j implies $E_{t-1+j}s_{t+j} = 0$ for all j, we can utilize the inventory policy in (B) to get $s_{t-1} = -\min\{0, \varepsilon_{t-1}\}$. Substituting this into the decision rule for y_t in (B) gives the following solutions that are exactly identical to that of Kahn (1987, equations 6, 7, 8):[14]

$$\begin{aligned} y_t &= \rho\theta_{t-1} + \min\{0, \varepsilon_{t-1}\}, \\ c_t &= \rho\theta_{t-1} + \min\{0, \varepsilon_t\}, \\ s_t &= -\min\{0, \varepsilon_t\}. \end{aligned}$$

It is easy to see that in this case, as long as $\rho > 0$, the variance of production exceeds the variance of sales (consumption) as is shown by Kahn (1987).

Here, production is more variable than sales because on one hand the existing stock available for sales, $y_t + s_{t-1}$, always targets the expected sales ($E_{t-1}c_t = \rho\theta_{t-1}$) under the stockout-avoidance motive, which makes production respond to changes in the expected demand one for one by replenishing inventories. On the other hand, production also keeps track of innovations in future demand (ε_t). Due to the information lag, however, production responds (optimally) only to innovations in the previous period, ε_{t-1}, which is correlated with expected demand ($E_{t-1}c_t$), rendering production to move more than one for one with changes in expected sales. Hence the variance of production exceeds the variance of sales:

$$\begin{aligned} var(y) &= var(\rho\theta_{t-1}) + \frac{1}{2}var(\varepsilon_{t-1}) + \frac{1}{2}cov(\rho\theta_{t-1}, \varepsilon_{t-1}) \\ &> var(\rho\theta_{t-1}) + \frac{1}{2}var(\varepsilon_t) \\ &= var(c) \end{aligned}$$

[14]Kahn (1987) has a positive constant k while we have a constant 0 appearing in the solutions because in our log-linear model the steady state values for the lower-case variables are zero.

where $\frac{1}{2}$ reflects the assumption that ε has a symmetric distribution around zero, suggesting that there exists a precautionary motive for production. The larger the serial correlation in demand shocks (ρ), the stronger such a motive.

The above analysis can be summarized in the following two propositions.

Proposition 2 *If ε_t has a symmetric distribution around zero, then the variance of production is less than the variance of sales and inventory investment is countercyclical.*[15]

Proof. See Appendix 2.■

Proposition 3 *In the case of (B), if demand shocks are serially correlated, the variance of production exceeds the variance of sales at all cyclical frequencies except at the zero frequency. Furthermore, inventory investment is always countercyclical at the high frequencies (2-4 quarters per cycle) but procyclical at any lower frequencies provided ρ is large enough.*

Proof. See Appendix 3.■

Proposition 3 is very powerful. It provides an rigorous mathematical proof as to why the stockout-avoidance mechanism is crucial for understating the phenomenon that inventory investment appears to be countercyclical at the high frequencies but procyclical at the business cycle frequencies. But proposition 3 is based on a unrealistic condition that realized shocks are greater than expectation (the conditional mean) with probability one, which is inconsistent with the rational expectations hypothesis. The following subsection shows, however, that there exists natural economic conditions such that the content in proposition 3 continues to hold in general equilibrium.

F. Results with Positive Storage Costs

With positive storage costs the shadow price (λ_t) is no longer a constant when demand shocks are negative.[16] Variable prices tend to decrease the volatility of consumption demand (sales). In

[15] The proposition also holds when ε_t has a nonzero mean $\bar{\varepsilon}$. In that case, the decision rules need to be adjusted to include $\bar{\varepsilon}$ when $E_{t-1}\theta_t$ is formed. $\bar{\varepsilon} = 0$ is simply a normalization without loss of generality.

[16] It can be easily shown that the shadow price follows the rule in the previous model (where $\phi = 0$, $\gamma = 0$, $\alpha = 1$, $\delta = 0$):

$$\lambda_t = \max\left\{0, \min\left\{\varepsilon_t, \varepsilon_t + \rho\theta_{t-1} - s_{t-1}\right\}\right\},$$

implying that $\lambda_t = \max\{0, \theta_t - s_{t-1}\}$ under case A and $\lambda_t = \max\{0, \varepsilon_t\}$ under case B. Under either case the shadow price can only go up if demand shocks are positive. It remains constant ($= 0$), however, if demand shocks are negative. This means that goods price is downward "sticky" under negative demand shocks. The fact that inventories can cause prices to be sticky has been pointed out by many people, e.g., see Amihud and Mendelson (1983) and Blinder (1982).

addition, storage costs discourage firms (or the planner) to use inventories to smooth production. Thus, if demand is expected to be low in advance, firms may opt to reduce production by cutting back employment rather than to keep production constant by holding inventories. These effects of storage costs on consumption (via price changes) and production reinforce with each other, making the variance of production more likely to exceed the variance of sales. This intuition is proved in the following propositions. To simplify the proofs without loss of insight, I consider only the special circumstance where $\gamma = 0$ and $\delta = 0$. Cases with more general parameter settings will be covered by numerical analysis.

Proposition 4 *With positive storage costs, the equilibrium decision rules for labor, production, consumption (sales), and the inventory stock are given by:*

$$n_t = \max\{n_{At}, n_{Bt}\}$$

$$y_t = \max\{\alpha n_{At}, \alpha n_{Bt}\},$$

$$c_t = \min\{\max\{\alpha n_{At} + s_{t-1}, \alpha n_{Bt} + s_{t-1}\}, \max\{\theta_t - \lambda_{At}, \theta_t - \lambda_{Bt}\}\}$$

$$s_t = \max\{0, \max\{\alpha n_{At} + s_{t-1} - \theta_t + \lambda_{At}, \alpha n_{Bt} + s_{t-1} - \theta_t + \lambda_{Bt}\}\}$$

where

$$\begin{aligned} n_{At} &= \rho\theta_{t-1} - s_{t-1} \\ n_{Bt} &= \xi\rho\theta_{t-1} - \psi s_{t-1} \\ \lambda_{At} &= \omega_1 s_{t-1} + \omega_2\theta_t + \omega_3\theta_{t-1} \\ \lambda_{Bt} &= \gamma_1 s_{t-1} + \gamma_2\theta_t + \gamma_3\theta_{t-1} \end{aligned}$$

where

$$\xi = \left[\frac{(\alpha-1)\left(1-\rho\beta-\rho+\rho\bar{\lambda}\beta\right)+\rho\phi}{(\alpha-1)\left(1-\rho\beta-\rho+\rho^2\beta\right)+\rho\phi}\right]\left[\frac{\phi}{(1-\alpha)\left(1-\bar{\lambda}\beta\right)+\phi}\right] \leq 1$$

$$\psi = \frac{\phi}{\phi + (1-\alpha)\left(1-\bar{\lambda}\beta\right)} \leq \xi \leq 1;$$

$$\begin{aligned} \omega_1 &= \phi\left[\frac{1-\alpha}{\bar{\eta}\beta - 1 - \phi}\right], \\ \omega_2 = \phi\left[\frac{(\alpha\rho-1)}{\bar{\eta}\beta-1-\phi}\right]&\left[\frac{(\rho\beta-1)(-1+\alpha)+\rho(\bar{\eta}\beta-1-\phi)}{(\rho\beta-1)(-1+\alpha)+\rho(\rho\beta-1-\phi)}\right], \\ \omega_3 = \phi\left[\frac{\alpha\rho(1-\rho)}{\bar{\eta}\beta-1-\phi}\right]&\left[\frac{(\rho\beta-1)(-1+\alpha)+\rho(\bar{\eta}\beta-1-\phi)}{(\rho\beta-1)(-1+\alpha)+\rho(\rho\beta-1-\phi)}\right], \\ \gamma_1 &= \phi\left[\frac{1-\alpha\psi}{\bar{\kappa}\beta-1-\phi}\right], \end{aligned}$$

$$\gamma_2 = \phi \left[\frac{(\alpha\rho\xi - 1)}{\bar{\kappa}\beta - 1 - \phi} \right] \left[\frac{(\rho\beta - 1)(-1 + \alpha\psi) + \rho(\bar{\kappa}\beta - 1 - \phi)}{(\rho\beta - 1)(-1 + \alpha\psi) + \rho(\rho\beta - 1 - \phi)} \right],$$

$$\gamma_3 = \phi \left[\frac{\alpha\xi\rho(1 - \rho)}{\bar{\kappa}\beta - 1 - \phi} \right] \left[\frac{(\rho\beta - 1)(-1 + \alpha\psi) + \rho(\bar{\kappa}\beta - 1 - \phi)}{(\rho\beta - 1)(-1 + \alpha\psi) + \rho(\rho\beta - 1 - \phi)} \right],$$

where $\bar{\lambda}, \bar{\eta}, \bar{\kappa}$ *are defined respectively as*

$$\bar{\lambda} = \frac{1}{2\beta(1 - \alpha)} \left\{ [(1 + \beta)(1 - \alpha) + \phi] - \sqrt{[(1 + \beta)(1 - \alpha) + \phi]^2 - 4\beta(1 - \alpha)^2} \right\} \in [0, 1]$$

$$\bar{\eta} = \frac{1}{2\beta} \left\{ (\beta - \alpha\beta + 1 + \phi) - \sqrt{(\beta - \alpha\beta + 1 + \phi)^2 - 4\beta(1 - \alpha)} \right\} \in [0, 1]$$

$$\bar{\kappa} = \frac{1}{2\beta} \left\{ (\beta - \alpha\beta\psi + 1 + \phi) - \sqrt{(\beta - \alpha\beta\psi + 1 + \phi)^2 - 4\beta(1 - \alpha\psi)} \right\} \in [0, 1]$$

Proof. See Appendix 4.■

It is easy to see that the decision rules reduce to those shown previously when $\phi = 0$, but are noticeably more complicated here with $\phi > 0$. The following proposition shows that the precondition needed for Kahn's (1987) result to hold in general equilibrium is precisely the condition $\phi > 0$.

Proposition 5 *When* $\alpha = 1$ *and* $\phi > 0$, *the variance of production exceeds the variance of sales and inventory investment is procyclical at the business-cycle frequencies but counter-cyclical at the high frequencies as long as* $\rho > 0$.

Proof. See Appendix 5.■

The following proposition shows that a different economic condition can also give rise to Kahn's (1987) result even when the marginal costs of production are not constant in output (i.e., $\alpha < 1$).

Proposition 6 *When* $\alpha < 1$, *it is possible to obtain the same results in proposition (5) if* ϕ *is sufficiently large.*

Proof. See Appendix 6.■

Proposition 7 *When* $\alpha < 1$ *and* $\phi < \infty$, *the variance of production is asymmetric with respect to demand shocks. Namely, firms adjust output more quickly following a positive demand shock than following a negative shock.*

Proof. See Appendix 7.■

To help understand this proposition, consider the optimal policy for employment with parameter values $\alpha < 1$ and $\phi < \infty$. The policy can be rewritten as

$$n_t = \begin{cases} E_{t-1}\theta_t - s_{t-1}, & \text{if } E_{t-1}\theta_t > a s_{t-1}; \\ \xi E_{t-1}\theta_t - \psi s_{t-1}, & \text{if } E_{t-1}\theta_t \le a s_{t-1}; \end{cases} \tag{3.18}$$

where $0 \leq \psi \leq \xi \leq 1$ and $a = \frac{1-\psi}{1-\xi} \geq 1$.[17] This optimal policy shows a number of things. First, it shows that firms increase production ($n_t > 0$) when expected demand is high at a different speed than they reduce production ($n_t \leq 0$) when expected demand is low. Namely, in case of a higher expected demand employment will respond to the increase in demand one for one (the top equation in 3.18); but in case of a lower expected demand employment responds to the decrease in demand less than one for one (the elasticity is $\xi < 1$ in the bottom equation in 3.18). In addition, there is an optimal target level of inventories determined by expected sales (demand). Namely, firms replenish inventories with unit elasticity in employment in case of a higher expected demand - for every one unit drop in s_{t-1} with respect to the target inventory stock employment will increase by one unit ($\frac{\partial n_t}{\partial s_{t-1}} = -1$ in the top equation in 3.18); but in case of a lower demand firms reduce inventories with an elasticity less than one - for each extra unit in s_{t-1} above the target inventory level employment will decrease only by $\psi < 1$ units (see bottom equation in 3.18). This suggests that the marginal cost associated with a stockout is different from the cost associated with holding one extra unit of inventory, with the former larger than the later.[18] Hence, production is more responsive to high demand than to low demand. This is so because holding inventories has a benefit - help smooth production (by reducing production costs) under demand uncertainty. Such an advantage of holding inventories disappears completely only in the limit when the marginal cost of production is constant (provided $\phi > 0$) or when the marginal cost of holding inventories (ϕ) is infinitely large. In these cases, the responses of production to expected demand shocks become symmetric, resulting in more variable production than sales.

G. A Calibrated Exercise

To facilitate proofs, proposition 6 is based on the limiting case when $\phi \to \infty$. In order to obtain Kahn's results, only finite values of ϕ are needed. This can be easily confirmed by a numerical simulation. Table 1 reports the variance of production and sales as well as the correlations between inventory investment and sales at both the high and the low frequencies when the parameters $\{\alpha, \beta, \phi, \delta, \gamma, \rho, y^*\}$ are calibrated to their empirical estimates. It shows that as long as ϕ is sufficiently large, Kahn's result continues to hold even if $\alpha < 1$. Following the standard RBC literature, I choose the time period in the model as a quarter, and set $\alpha = 0.7$ according to labor's income share in output in the US, $\beta = 0.99$ for the time discount factor, $\gamma = 0$ according

[17] The threshold value of the expected demand that determines the action of n_t is derived by using the non-negativity constraint on inventories, $E_{t-1}s_t \geq 0$ in case B in Appendix 4. This gives $a = \frac{1-\psi}{1-\xi}$.

[18] Krane (1994) has made a similar point using a partial equilibrium model.

to the indivisible labor literature (e.g., see Hansen, 1985),[19] $\delta = 0.025$ for the depreciation rate of inventories (which amounts to 10 percent a year). To calibrate steady-state output level y^*, I use the aggregate profit rate or output-capital ratio as a measure of the productivity parameter A, which is about 10 percent.[20] Assume that the economy is a per-worker economy and that the fraction of hours worked in a week is 0.2, then $y^* = 0.1\,(0.2)^{0.7} \approx 0.03$. The persistence parameter for demand shocks (ρ) is set at 0.9. The variance of the innovation ε_t is normalized to one.

Table 2 shows that depending on the size of the storage cost (ϕ), the variance of production can be either larger or smaller than the variance of sales, and inventory investment can be either procyclical or countercyclical with respect to sales. In particular, given that the costs of production are convex ($\alpha = 0.7$), the size of the storage cost for holding inventories is crucial for determining the relative volatilities of production and sales as well as the correlations between inventory investment and sales. When the storage cost is small (e.g., $\phi = 0.1$), the production-smoothing motive prevails, hence the variance of production is less than the variance of sales at both the high frequencies and the business-cycle frequencies, and inventory investment is countercyclical also across both high and low frequencies. However, when the marginal storage cost is sufficiently large (e.g., $\phi = 10$), the stockout-avoidance motive starts to prevail. Consequently the variance of production exceeds the variance of sales at both high and low frequencies and inventory investment becomes procyclical at the business cycle frequencies.[21]

The conclusion is that with standard concave production function, it is impossible to guarantee that the variance of production exceeds the variance of sales despite the stockout-avoidance motive and serially correlated demand shocks, unless economic conditions are such that the motives for production smoothing are substantially weakened. Only then do firms find it optimal to cut production sufficiently downward during economic downturns (in responding to low demand), rendering production more volatile than sales and inventory investment procyclical, hence giving rise to the observation in the introduction of the paper, "firms tend to over-produce during booms and over-cut output during recessions". One of Kahn's (1987) important insights is to show that economic conditions weakening the motives for production smoothing do exist. Kahn's condition requires the marginal cost of production be constant ($\alpha = 1$). I have shown another one: the

[19] Increasing γ has a very similar effect to that of decreasing α, as both amount to increase the convexity of production costs.

[20] For example, in a standard endogenous growth model (the AK model), A measures the rate of return to capital.

[21] In this simple model without fixed-capital, however, it is impossible to have $\sigma_y > \sigma_c$ only at the business cycle frequencies but not at the high frequencies. In other words, production is either more volatile or less volatile than sales at all frequencies. Such a prediction runs counter to the fact showing in table 1. As will be shown shortly, introducing capital investment can resolve this problem.

marginal storage cost for holding inventories (ϕ) be sufficiently large. But neither condition is likely to hold well in reality across various industries. In addition, these models tend to predict more variable production than sales at all frequencies. Such a prediction is inconsistent with the data where production is smooth relative sales at the high frequencies. There exist a third alternative, however, which is the existence of a productive asset that yields higher rate of return than inventories in the long run, so that firms' incentives for using inventories to smooth production are diminished. With a stockout-voidance motive and a diminished inventive for production smoothing, inventory investment is more likely to be procyclical, rendering possible that the variance of production exceeds the variance of sales. To investigate this we now turn.

4 A Complete Model with Capital

Assume that capital-investment decisions must be made in advance for at least one period, then capital investment will not be able to respond to demand shocks instantaneously. Due to such an information friction, fixed capital is not as liquid as inventories as a means to buffer shocks in the short run. Hence it is optimal to hold inventories in the very short run during which capital investment cannot adjust. In the longer run, however, holding inventories is inefficient. Therefore, it is rational to set $E_{t-j}s_t = 0$ where $j \geq 0$ denotes the time lags it takes for capital investment to respond to time t shocks. In this paper, I assume $j = 1$ (i.e., one quarter).

The representative agent's problem is to choose sequences of consumption $\{C_t\}_{t=0}^{\infty}$, hours to work $\{N_t\}_{t=0}^{\infty}$, savings in terms of capital investment $\{K_{t+1}\}_{t=0}^{\infty}$ and inventory investment $\{S_t\}_{t=0}^{\infty}$ to solve:

$$\max_{\{K_{t+1},N_t\}_{t=0}^{\infty}} E_{t-1}\left\{\max_{\{C_t,S_t\}_{t=0}^{\infty}} E_t\left\{\sum_{t=0}^{\infty}\beta^t\left\{\Theta_t \log C_t - a\frac{N_t^{1+\gamma}}{1+\gamma}\right\}\right\}\right\}$$

subject to

$$C_t + [S_t - (1-\delta_s)S_{t-1}] + [K_{t+1} - (1-\delta_k)K_t] = AK_t^{1-\alpha}N_t^{\alpha} - \frac{\phi}{2}S_t^2, \tag{4.1}$$

$$S_t \geq 0; \tag{4.2}$$

and $S_{-1} \geq 0, K_0 \geq 0$ given; where K_t denotes the existing capital stock (I have followed the convention to denote K_t as the existing capital stock). The model is identical to the one studied previously except there is a fixed capital asset here. The first order conditions are given by:

$$E_{t-1}\left\{aN_t^{\gamma} - \Lambda_t \alpha A K_t^{1-\alpha}N_t^{\alpha-1}\right\} = 0$$

$$E_{t-1}\left\{\Lambda_t - \beta\Lambda_{t+1}\left[(1-\alpha)AK_{t+1}^{-\alpha}N_{t+1}^{\alpha} + (1-\delta_k)\right]\right\} = 0$$

$$\frac{\Theta_t}{C_t} = \Lambda_t$$

$$\Lambda_t (1 + \phi S_t) = \beta(1 - \delta_s) E_t \Lambda_{t+1} + \Gamma_t$$

$$C_t + [S_t - (1 - \delta_s) S_{t-1}] + [K_{t+1} - (1 - \delta_k) K_t] = A K_t^{1-\alpha} N_t^{\alpha} - \frac{\phi}{2} S_t^2$$

$$S_t = 0 \quad \text{if and only if } \Gamma_t > 0$$

$$S_t \geq 0 \quad \text{if and only if } \Gamma_t = 0$$

plus two transversality conditions,

$$\lim_{T \to \infty} \beta^T \Lambda_T K_{T+1} = 0$$

$$\lim_{T \to \infty} \beta^T \Lambda_T (1 + \phi S_T) S_T = 0,$$

where Λ_t and Γ_t are Lagrangian multipliers associated with (4.1) and (4.2) respectively.[22]

A. Solution Method

I use the same linearization method outlined in the previous section to find equilibrium decision rules near the steady state. The linearized first-order conditions are given by:

$$E_{t-1} \{(1 + \gamma - \alpha) n_t - \lambda_t - (1 - \alpha) k_t\} = 0$$

$$E_{t-1} \{\lambda_{t+1} + (1 - \beta (1 - \delta_k)) [\alpha n_{t+1} - \alpha k_{t+1}] - \lambda_t\} = 0$$

$$\theta_t - c_t = \lambda_t$$

$$\lambda_t + \phi s_t = E_{t+1} \lambda_{t+1} + \pi_t$$

$$(1 - \mu) c_t + \mu \left[\frac{1}{\delta_k} k_{t+1} - \frac{1 - \delta_k}{\delta_k} k_t\right] + \frac{1}{y^*} [s_t - (1 - \delta_s) s_{t-1}] = (1 - \alpha) k_t + \alpha n_t$$

$$s_t = 0 \quad \text{if and only if } \pi_t > 0$$

$$s_t \geq 0 \quad \text{if and only if } \pi_t = 0$$

where μ is the steady-state share of capital investment in output and $(1 - \mu)$ the steady-state share of consumption in output. Note that the steady state share of inventory investment in output is zero in this model, as in the model studied previously. The steps for computing equilibrium decisions rules in this model are very similar to that outlined previously. The only difference is that in the current model both employment and capital investment decisions are made based on information available in time period $t - 1$, hence the possibility of case B ($E_{t-1} s_t > 0$) never arises. Namely, regardless the magnitude and sign of expected demand shocks ($E_{t-1} \theta_t$), capital

[22] I assume that a non-negativity constraint on the capital stock (K_{t+1}) never binds. This is valid near the steady state.

investment is chosen such that expected inventory stock is zero. Therefore in solving for the decision rules of employment and capital investment, the first-order condition with respect to inventory stock (s_t) is ignored (since it is irrelevant due to the expected-rate-of-return dominance by fixed capital) and the optimality condition, $E_{t-1}s_t = 0$, is imposed.[23]

B. Calibration

I follow the existing RBC literature by calibrating the structural parameters of the model. In particular, I choose the time period as a quarter, and set the labor's income share $(1-\alpha) = 0.7$, the inverse labor supply elasticity parameter $\gamma = 0$ (Hansen's 1985 indivisible labor), the rate of capital depreciation $\delta_k = 0.025$, the rate of inventory depreciation $\delta_s = 0.025$, the time discount factor $\beta = 0.99$, the steady-state technology coefficient $A = 0.1$ (implying a 10 percent rate of return to capital in an endogenous AK growth model), the steady-state fraction of hours worked in a week is calibrated at $n = 0.2$. These parameters imply a steady-state saving ratio (fixed investment-to-output ratio) $\mu = 0.2$, and a steady-state output level $y^* = 0.02$. Although the parameter values for A, δ_s, and hence y^* are somewhat arbitrary, the predictions of the model are not sensitive to these parameters at all. Finally, the persistence parameter of consumption shock (ρ) is important for determining whether inventory investment in the model is countercyclical or procyclical, and it is also important for determining the variance of production relative to sales. Empirical estimates (e.g., see Baxter and King 1991, Wen 2002) show that ρ is greater than 0.9 but less than one. I take the value $\rho = 0.97$ from Baxter and King (1991), since the model is closer to theirs than to any others in the RBC literature. There is very little empirical evidence on the parameter (ϕ) for storage costs. Hence I leave ϕ as a free parameter for robustness analysis.

C. Predictions for Inventory Cycles

Table 3 reports the relative standard deviations of production with respect to total sales (production minus inventory investment) and correlations between inventory investment and total sales when the storage cost parameter ϕ takes different values. The statistics shown are the means of predicted moments based on 100 simulations with sample length of 140 (standard errors are in parentheses). Table 3 shows that the general equilibrium model with both inventory investment and capital investment is capable of explaining the stylized facts of inventory cycles documented in table 1. Namely:

[23] The first-order condition with respect to s_t will be relevant when solving for the time-t decision variables such as $\{c_t, s_t\}$.

1) At the high frequencies (1st column), predicted production is less volatile than sales when the marginal storage cost is small or not too large ($\phi \in [0, 2)$). The predicted ratio of standard deviations between output and sales, for example, is around 0.9 when $\phi = 0$ and 0.95 when $\phi = 1$. These predictions are fully consistent with the data. As is also the case in the data, there exist possibilities where this ratio may exceed one at the high frequencies (e.g., the United States and Finland). Here, such possibilities are captured by the parameter ϕ. When ϕ is large enough (e.g., $\phi = 5$), the predicted volatility ratio is 1.17 (this can also happen when ρ is larger, see below).

2) At the high frequencies (2nd column), predicted inventory investment is strongly countercyclical with respect to sales. The predicted correlation between inventory investment and sales, for example, is -0.86 for $\phi = 0$, and this holds for all possible values of ϕ without exception.

3) In stark contrast, at the business cycle frequencies (3rd column) predicted production is more volatile than sales. The predicted ratio of standard deviations between output and sales, for example, is 1.07 with a very tight standard error of 0.001 for all the possible parameter values of ϕ considered.

4) At the business cycle frequencies (4th column), predicted inventory investment is positively correlated with sales. The predicted correlation between inventory investment and sales exceeds 0.3 with tight standard errors for all possible parameter values of ϕ considered.

These predictions are very similar to those of the previous model without capital, but with two important exceptions: First, in the current model the variance of production can exceed the variance of sales at the business-cycle frequencies even when $\phi = 0$. Second, the variance of production can be smaller than the variance of sales at the high frequencies but simultaneously larger than that of sales at the business-cycle frequencies for $\phi < 2$. These results are impossible for the model without capital. The intuition for the current model to achieve these nice results is as follows. Consider the case when the demand shock is expected to be negative. In the previous model, production is downward sticky and firms opt to plan for holding inventories so as to smooth production (i.e., $E_{t-1}s_t > 0$). Hence a large value of α or ϕ is needed in order to weaken the motive for production smoothing. In the current model, however, inventory investment is not efficient when the option for capital investment exists at the time when capital investment decision is made. Hence $E_{t-1}s_t = 0$ regardless demand shocks being positive or negative. Firms can consider using the capital stock to smooth production instead. This, however, implies that capital investment need to increase (so as to buffer the expected decrease in consumption) in a situation when the marginal utility of consumption is expected to be low. This is feasible but may not be optimal, however, if the shocks are highly serially correlated, because then the benefit associated with increasing production capacity (capital stock) in a situation when future marginal utilities of consumption are expected to be low for many periods to come may not outweigh the benefit of reducing labor costs (in terms of leisure) associated with a cutback in production. Therefore, it may be in the firm's (or planner's) best interest to have fixed investment move together with (rather than against) consumption demand, making production more volatile rather than being smooth.[24] Consequently, with serially correlated demand shocks production may cease to be downward sticky, unlike the case in the previous model without capital. To confirm this intuition, table 4 reports predicted moments for different values of the persistence parameter ρ. It shows that the more persistent of the shocks, the larger the correlation between consumption and capital investment, hence the more likely for the variance of production to exceed the variance of sales. For example, when $\rho = 0.5$, capital investment and consumption are negatively correlated (-0.6), hence the variance of production is less than the variance of sales at both the high frequencies and the business-cycle frequencies. But when $\rho = 0.99$, for example, capital investment is highly positively correlated with consumption (0.85), hence production becomes more volatile than sales at both the high frequencies and the business-cycle frequencies (this perhaps explains why some countries in table 1, such as the US and Finland, have larger variances of production relative to sales at both high and low frequency intervals). When ρ takes values in between, we obtain the normal cases where production is more variable than sales only at the business-cycle frequencies.

Quantitatively speaking, however, the model has two obvious shortcomings (obvious in both table 3 and table 4). It tends to generate too strong a negative correlation between inventory investment and sales at the high frequencies compared to what is in the data (e.g., -0.86 versus -0.4). And it generates a volatility ratio between output and sales that is not large enough at the business cycle frequencies compared to what is in the data (e.g., 1.07 versus 1.38). These two shortcomings are quite robust to parameter values. Considering the simplicity of the model, however, its performance is extraordinary.

[24] This can be illustrated using the following identity. Since $y = z + \Delta s = (c + \Delta k) + \Delta s$, we have:

$$var(y) = var(z) + var(\Delta s) + 2cov(z, \Delta s)$$

where

$$cov(z, \Delta s) = cov(c, \Delta s) + cov(\Delta k, \Delta s).$$

Note that $var(y) > var(z)$ if $cor(z, \Delta s) > 0$. Given that $cor(c, \Delta s) > 0$ as shown in the model without capital, $cor(\Delta k, c) > 0$ then implies $cor(\Delta k, \Delta s) > 0$, and hence $cor(z, \Delta s) > 0$.

D. Why is Inventory Investment so Volatile?

Another stylized fact puzzling economists is the extremely volatile changes in inventory investment relative to changes in GDP observed in the data. The literature reports that the drop in inventory investment has accounted for 87 percent of the drop in total output during the average postwar recessions in the US (Blinder, 1991). Based on such a surprisingly large contribution to GDP fluctuations from inventory investment, Blinder concludes that business cycles are essentially inventory fluctuations. Careful re-examinations of the data show, however, that unlike GDP or capital investment, the bulk of volatility in inventory investment is concentrated around the very high cyclical frequencies (2-3 quarters per cycle) rather than around the lower frequency interval such as the business-cycle frequencies (8-40 quarters per cycle).

To document this feature of inventory investment, define a relative volatility statistic (r) as the ratio between movement (relative to GDP) in inventory investment and movement (relative to trend) in GDP as

$$r = \frac{\frac{1}{T}\sum_{t=1}^{T}\left|\frac{\Delta S_t}{Y^*}\right|}{\frac{1}{T}\sum_{t=1}^{T}\left|\log Y_t - \gamma t\right|},$$

where ΔS denotes inventory investment or changes in the inventory stock, Y denotes GDP, $\frac{\Delta S_t}{Y^*}$ denotes inventory investment normalized by the median of GDP (Y^*), $(\log Y_t - \gamma t)$ denotes percentage deviations of GDP relative to a deterministic long-run trend, and $|\cdot|$ denotes the absolute-value operator. Thus, $\left|\frac{\Delta S_t}{Y^*}\right|$ measures the volatility of inventory investment relative to GDP and $|\log Y_t - \gamma t|$ measures the volatility of GDP relative to trend. The statistic r therefore measures the changes in inventory investment as a fraction of GDP with respect to changes in GDP around trend. For example, $r = 1$ implies roughly that for one percent change in inventory investment as a fraction of GDP there is associated one percent change in GDP relative to trend. Hence the larger the r, the more contributions inventory investment has to GDP fluctuations. To compare such contributions across different cyclical frequencies, the band-pass filter is applied to the time series $\{\frac{\Delta S_t}{Y^*}\}_{t=1}^{T}$ and $\{\log Y_t - \gamma t\}_{t=1}^{T}$ so that the r statistics can be estimated for data in different frequency bands.

Table 5 reports the r statistics for the same OECD economics considered in table 1. It shows that at the high frequency interval $\omega \in [2/3\pi, \pi]$, which corresponds to movements between 2-quarter cycle to 3-quarter cycle, the r statistic is quite large, ranging from 0.55 to 1.51, indicating very large contributions to GDP fluctuations from inventory investment (the sample mean of r for the 11 countries listed is 1.03). In contrast, at the business-cycle frequency interval $\omega \in [0.05\pi, 0.25\pi]$, which corresponds to movements from 8-quarter cycle to 40-quarter cycle, the estimated contributions of inventory investment to GDP fluctuations are much smaller, with the

r statistic ranging from 0.21 to 0.82 (the sample mean is 0.49). Notice that although the r statistics change quite a lot across individual countries, for every country in the sample r_H (the statistic at the high frequencies) is always larger than r_L (the statistic at the lower business-cycle frequencies).[25]

To better appreciate such a big difference in contribution across frequencies, define the ratio, $\varkappa = \frac{r_H}{r_L}$, as an index for the relative contribution change across the high and the low frequencies. The last column in table 5 shows that on average inventory investment contributes more than twice as much to GDP fluctuations at the high frequencies as it does at the business-cycle frequencies (the sample average of $\varkappa$ is 2.2), indicating that the bulk of variance in inventory investment is concentrated at the high frequencies. The smallest $\varkappa$ index comes from Australia (1.3), the largest $\varkappa$ index is from Canada (3.4). The $\varkappa$ index for the United States is 2.8.[26]

Why does inventory investment account significantly more for GDP fluctuations at the high frequencies than at the business-cycle frequencies? The answer perhaps lies in the fact that inventory stock as a buffer against consumption shocks is far more important in the very short run (high frequencies) than it is in the longer run (lower frequencies), because in the longer run capital investment can substitute inventory investment as a buffer for consumption against preference shocks, consequently the buffer-stock role of inventories diminishes in time once capital investment becomes capable of adjusting to changes in consumption demand.

This intuition is confirmed by the general equilibrium model. The bottom rows of table 5 (numbers shown are the means of 100 simulations with sample length of 140, standard errors are in parentheses) show that the predicted r statistic is 1.65 at the high frequencies but only 0.14 at the business-cycle frequencies (when $\phi = 0$), indicating that in an environment where employment and capital investment are not able to adjust to consumption shocks in the short run, inventory investment will appear to be more volatile relative to output at the 2-3 quarter frequencies, since only inventory investment can absorb the full impact of consumption-demand shocks in the very short run. The model, however, tends to over-predict such volatility differences across frequencies. The predicted $\varkappa$ index is 12, about 6 times larger than what is observed in the data. This quantitative inconsistence between the model and the data cannot be attributed to storage costs which affect the variance of inventory investment. As the bottom rows in table 5 show, increasing the marginal storage costs (ϕ) does not help bringing down the $\varkappa$ index. In fact,

[25] This is also true for both aggregated OECD data and aggregated European country data. For example, for OECD countries as a whole, $r_H = 0.67$ and $r_L = 0.45$. For the European countries as a whole, $r_H = 0.74$ and $r_L = 0.36$.

[26] The $\varkappa$ index is 2.1 for OECD countries as a whole and is 1.5 for European countries as a whole.

$\varkappa$ is also robust to other parameters in the model including ρ, suggesting that the reasons as to why the model over-predicts the $\varkappa$ index have to be found somewhere else.

5 Explaining the Business Cycle

Qualitatively speaking, the inventory cycle is well explained by general equilibrium theory with consumption-demand uncertainty alone. Inventories play a key role in buffering consumption against unanticipated changes in preferences. In a decentralized economy, changes in preferences translate into changes in demand and prices. Profit-maximizing firms opt to hold inventories so as to avoid losses of opportunity for sale despite flexible prices. Such a stockout-avoidance motive is the key for understanding the observed inventory behavior. Productivity shocks or nonconvex costs thus appear to be unnecessary for understanding inventory fluctuations.

An important remaining question is, can consumption demand shocks also explain the business cycle? This section takes up this question along two lines. First, I document a link between inventory fluctuations and the business cycle by showing that the economic condition for the existence of inventories is intimately linked to some prominent features of the business cycle. Then I simulate the simple general equilibrium model driven by consumption shocks alone to see if its predictions match some general features of postwar US business cycles emphasized by the literature.

A. Testing the Information Structure

An assumption pivotal to the success of the general equilibrium model in accounting for inventory fluctuations is the information structure that decisions about production and capital investment must be taken before demand uncertainty is resolved. If production and capital investment can adjust instantaneously to respond to demand shocks, not only does the stockout-avoidance motive for holding inventories disappear, but also does the very rationale for the existence of inventories at the first place due to the rate-of-return dominance by capital investment over inventory investment. How realistic is the information structure? This information structure has a striking implication for the business cycle: If consumption demand uncertainty is indeed the main source of the business cycle and if decisions about production and capital investment are indeed made in advance before demand uncertainty is resolved, then consumption must appear to Granger-cause production (employment) and capital investment, but not vise versa. To see this, note that consumption, output, and capital investment in the model follow the following decision rules:

$$
\begin{aligned}
c_t &= c\left(k_t, s_{t-1}, \theta_t, \theta_{t-1}\right), \\
y_t &= y\left(k_t, s_{t-1}, \theta_{t-1}\right), \\
i_t &= i\left(k_t, s_{t-1}, \theta_{t-1}\right);
\end{aligned}
$$

where the most recent information about demand shocks θ_t appears in consumption but not in output and fixed investment. Substituting out the endogenous state variables $\{k_t, s_{t-1}\}$ in these decision rules using the equilibrium law of motion of the model (in the linearized version):

$$
\begin{pmatrix} k_t \\ s_{t-1} \end{pmatrix} = (I - ML)^{-1} R \begin{pmatrix} \theta_{t-1} \\ \theta_{t-2} \end{pmatrix},
$$

where I is a 2×2 identity matrix, M and R are 2×2 coefficient matrices and L is the lag operator, the decision rules for $\{c, y, i\}$ can be rewritten as

$$
\begin{aligned}
c_t &= c\left(c_{t-1}, c_{t-2}, \theta_t, \theta_{t-1}, \ldots\right), \\
y_t &= y\left(y_{t-1}, y_{t-2}, \theta_{t-1}, \theta_{t-2}, \ldots\right), \\
i_t &= i\left(i_{t-1}, i_{t-2}, \theta_{t-1}, \theta_{t-2}, \ldots\right).
\end{aligned}
$$

These equilibrium policy rules imply that consumption in the preceding period helps predicting output and investment in the current period even after the history of output and investment is taken into account. This is so because c_{t-1} contains information for demand shock θ_{t-1} that is useful for predicting y_t and i_t but is missing in the past history of y_t and i_t, $\{y_{t-1}, y_{t-2}, \ldots, i_{t-1}, i_{t-2}, \ldots\}$, which contains information for the history of demand shocks only up to θ_{t-2}.

Note that the time-t information variable θ_t will not appear in the consumption decision rule if there is no inventories in the model, because in that case consumption is not capable of reacting to period-t shock (θ_t) given the resource constraint, $c_t = y_t - i_t$, where both production (y_t) and capital investment (i_t) are predetermined before θ_t taking place. Consequently, aggregate consumption will not appear to lead nor to Granger cause output and investment if the economy does not hold inventories, indicating that the existence of inventories is crucial for understanding the causal aspect of the business cycle.[27]

This section tests such an implication using the US data.[28] To find the causal relations among aggregate consumption, output, and capital investment, I first estimate the following sets of equations by ordinary least squares:

[27] Wen (2001) shows that consumption fails to Granger cause output and capital investment in a general-equilibrium model driven by preference shocks without inventories.

[28] Test results similar to the US are also found in aggregate European data, but not for all individual countries.

$$\begin{aligned}
\Delta y_t &= f(\Delta y_{t-1}, \Delta y_{t-2}), \\
\Delta y_t &= f(\Delta y_{t-1}, \Delta y_{t-2}, \Delta c_{t-1}); \\
\Delta i_t &= f(\Delta i_{t-1}, \Delta i_{t-2}), \\
\Delta i_t &= f(\Delta y_{t-1}, \Delta y_{t-2}, \Delta c_{t-1});
\end{aligned} \tag{5.1}$$

where Δy is growth in real GDP, Δi is growth in real fixed-capital investment, and Δc is growth in real consumption of non-durable goods and services.[29] A variable x is said to be "Granger causing" a variable y when a prediction of y on the basis of its past history can be improved by further taking into account the previous period's x (See Granger 1969). Estimating (5.1) gives the following results ($t-$values are in parentheses):

$$\begin{array}{llllll}
\Delta y_t = & 0.007 & -0.00003t & +0.25\Delta y_{t-1} & +0.10\Delta y_{t-2}; & \\
& (3.87)^{***} & (-1.59) & (3.01)^{***} & (1.26) & \\
\Delta y_t = & 0.001 & -0.00001t & +0.06\Delta y_{t-1} & +0.05\Delta y_{t-2} & +0.83\Delta c_{t-1}; \\
& (0.46) & (-0.37) & (0.68) & (0.63) & (4.67)^{***}
\end{array} \tag{5.2}$$

$$\begin{array}{llllll}
\Delta i_t = & 0.007 & -0.00002t & +0.39\Delta i_{t-1} & +0.20\Delta i_{t-2}; & \\
& (1.84) & (-0.60) & (4.73)^{***} & (2.40)^{***} & \\
\Delta i_t = & -0.005 & +0.000018t & +0.288\Delta i_{t-1} & +0.224\Delta i_{t-2} & +1.18\Delta c_{t-1}; \\
& (-0.93) & (0.43) & (3.33)^{***} & (2.79)^{***} & (3.09)^{***}
\end{array} \tag{5.3}$$

These results lead to the following conclusions. First, regression results in (5.2) suggest that past growth in consumption has a significant effect on current output growth even after past

This could be due to the low frequency (quarterly) data used. Data at monthly frequency may resolve this problem, but monthly aggregate data are rarely available for European or OECD countries. Due to limit in space, results for other countries are not discussed here.

[29] The data used are quarterly US data (1960:1 - 1996:3). Aggregate output is measured as real GDP. Aggregate consumption is measured as total consumption of nondurable goods and services. Aggregate investment is measured as business fixed investment. In terms of CITIBASE labels, these variables are named GDPQ, GCNQ+GCSQ, GINQ. The growth rates are formed as log differences. Since in practice GDP is measured as total expenditure that includes both consumption and inventory investment and is therefore not the same as production defined in the model in terms of information content, it is thus better to use employment rather than GDP in testing the Granger causality relations. It turns out, however, that the results are very similar regardless GDP or employment is used (but for some OECD or European countries this distinction maybe important).

history of output growth is taken into account. The coefficient for consumption growth, Δc_{t-1}, has a $t-$value of 4.67, far exceeds the 5% critical value of 1.96. On average, one percent increase in consumption growth in the preceding period induces 0.83 percent faster growth in output (the standard error is 0.18). In fact, consumption growth is so important in predicting future output growth such that none of the dependent variables in the original regression remains significant after past consumption growth is taken into account. All information contained in the past history of output growth is captured by consumption growth. The R^2 of the regression is increased by 200% after lagged consumption growth is taken into account.[30]

The situation is very similar for capital investment as is shown in regression results (5.3). Namely, past consumption growth has a strong explanatory power for current investment growth even after the history of investment growth is taken into account. The coefficient of past consumption growth in (5.3) suggests that one percentage increase in consumption growth in the preceding period induces current investment to grow 1.18 percentage faster.

For the reversed questions, whether past output growth or investment growth has an effect on current consumption growth given the history of consumption growth, I obtained the following results ($t-$values are in parentheses):

$$\begin{array}{llllll}
\Delta c_t = & 0.007 & -0.00003t & +0.28\Delta c_{t-1} & +0.07\Delta c_{t-2}; & \\
 & (5.81)^{***} & (-3.24)^{***} & (3.40)^{***} & (0.86) & \\
\Delta c_t = & 0.008 & -0.00003t & +0.24\Delta c_{t-1} & +0.04\Delta c_{t-2} & +0.05\Delta y_{t-1}; \\
 & (5.89)^{***} & (-3.31)^{***} & (2.62)^{***} & (0.44) & (0.99) \\
\Delta c_t = & 0.007 & -0.0003t & +0.290\Delta c_{t-1} & +0.083\Delta c_{t-2} & -0.0087\Delta i_{t-1}; \\
 & (5.55)^{***} & (-3.14)^{***} & (3.38)^{***} & (0.97) & (-0.50)
\end{array} \tag{5.4}$$

Regression results in (5.4) suggest that neither past output growth nor past investment growth has explanatory power for current consumption growth. The history of consumption growth is the best predictor for current and future consumption growth. In fact, including past investment and output growth into the information set has virtually zero effect on the original regression coefficients for consumption growth, nor on the R^2s. These results suggest a one-way causal relationship between consumption and output as well as capital investment. Namely, consumption growth in the preceding period Granger causes both output growth and investment growth in the current period, but not vise versa.[31]

[30] The results are very similar when more lags are included in the regressions.

[31] The fact that consumption appears to lead the business cycle is well known (e.g., see Benhabib and Wen

With regard to relationships between inventory fluctuations and the business cycle, the existing literature often emphasizes the issue of whether inventories are stabilizing or destabilizing to the economy. The test result provided here tells us another link between inventories and the business cycle. That is, the very condition for the existence of inventories is intimately related to how the business cycle behaves. If consumption uncertainty is indeed the ultimate cause of economic fluctuations, then the fact that firms use inventories to buffer such uncertainty implies that information frictions exist, which will surely result in consumption appearing to cause the business cycle. Without inventories, note this again, the model predicts that consumption does not Granger cause output and investment in spite of preference shocks, because consumption will not be able to move when everything else in the resource constraint (production and capital investment) is fixed at the time of shocks unless there exist inventories to allow consumption to move. If consumption cannot respond to time t shocks, then it cannot appear to cause output and investment.

B. Predicting Postwar US Business Cycles

There are three prominent features of the business cycle that has been the focus of the RBC literature: the positive comovements among output, consumption, employment and capital investment; the positive serial correlation in each of these variables; and the phenomenon that consumption appears to be smooth relative to income whereas capital investment appears to be more volatile than output (e.g., see Lucas 1977, Kydland and Prescott 1982, and Long and Plosser 1983). Standard RBC models explain these two features of the business cycle by relying on technology shocks. Consumption shocks are considered inconsistent with these business cycle facts because increases in autonomous consumption spending in a general equilibrium model tend to increase leisure (hence decrease labor supply) and crowd out capital investment, resulting in negative comovements among consumption, investment and output. In addition, when the source of shocks is from preference, consumption tends to be more volatile than savings, resulting in capital

2000). But a time series leading another does not imply Granger causality (see Sargent, 1987). The notion that consumption Granger-causes the business cycle is not completely unfamiliar either. Hall (1978), for example, tested the hypothesis that consumption follows a random walk, implying that nothing else except the history of consumption itself has predictive power on future consumption. Although the random walk hypothesis has not been fared well empirically (for example, consumption growth is strongly serially correlated), and this hypothesis does not imply that consumption Granger causes the business cycle, but Hall's test does show that consumption is not Granger caused by output in the economy, suggesting that consumption leads the business cycle. Using VAR impulse analysis, Blanchard (1993) and Cochrane (1994) also show that consumption shocks appear to be the major causes of the business cycle. These analyses are not equivalent but closely related to the Granger causality test performed here.

investment being smoother than output and consumption in the model. That this argument is incorrect has been proved recently by Wen (2002). Using a standard RBC model without inventories, Wen (2002) shows that consumption (preference) shocks can generate positive responses from employment and output as well as volatile and positive movements in capital investment if such shocks are highly serially correlated. The intuition is that with persistent consumption (preference) shocks, the marginal utilities of current consumption and future consumption are complementary, hence not only current consumption but also future consumption tend to go up. But the only way to achieve this is to increase both current production and current investment so that future production capacity can meet the higher expected future demand. The consequence is that consumption, employment, output and investment comove together under serially correlated preference shocks. In addition, since the marginal utilities of future consumption may outweigh that of current consumption when the shocks are highly persistent, savings may become more volatile than consumption, leading to relatively smooth consumption series compared to income and capital investment. I show here that the same intuition applies to the model with inventories.

Table 6 reports the estimated second moments of the US economy (first row) and the predicted second moments of the model (M1) with respect to output, consumption, capital investment, and hours.[32] The first column reports the ratios of standard deviations between $\{c, i, n\}$ and output (y), the second column reports the contemporaneous correlations between $\{c, i, n\}$ and y, and the third column reports the autocorrelations for each of these variables considered. Table 6 shows that the model qualitatively captures the relatively low volatility of consumption and the relatively high volatility of capital investment with respect to output. The predicted ratio of standard deviation of consumption relative to output is 0.98, and this ratio for investment is 1.45. The model also correctly predicts the positive correlations of consumption, investment and hours with output (which are $0.81, 0.37$, and 1.0 respectively), as well as the positive autocorrelations of these variables (which are $0.66, 0.72, 0.32$, and 0.67 respectively). Thus, qualitatively speaking, consumption shocks are capable of explaining the most prominent features of the business cycle, in addition to explaining the inventory cycle.

Quantitatively speaking, however, the model falls short in matching the data. In particular, the model tends to over-predict the volatility of consumption (0.98 in the model versus 0.79 in the data) and under-predict the volatility of investment (1.45 in the model versus 3.18 in the data). The model also tends to under-predict the correlation between investment and output (0.37 in the model versus 0.89 in the data) as well as the autocorrelation of investment (0.32 in the model

[32] Both the data and the simulated series are filtered by the H-P filter. To match the variance of US output, the variance of ε_t is set at $\sigma_\varepsilon^2 = 0.0055$.

versus 0.89 in the data). This may be the reason why the model also falls short quantitatively in predicting the inventory behavior discussed previously. Wen (2002) shows that allowing for habit formation in consumption can significantly reduce the volatility of consumption and increase the volatility of investment under preference shocks because habit formation effectively increases the persistence of preference shocks endogenously. The current model does not have habit formation, but as a suggestive exercise to stimulate further research, I reset the persistence parameter to $\rho = 0.99$, and reset capital's share $\alpha = 0.2$ so as to increase also the marginal product of labor. The bottom row in table 6 (M2) shows that predicted consumption volatility is reduced significantly to 0.68 and predicted investment volatility is increased significantly to 2.57. These numbers are much closer to the data. As a consequence, predicted correlation between investment and output as well as autocorrelation for investment are also improved significantly (the correlation increases from 0.37 to 0.85 and the autocorrelation increases from 0.32 to 0.70).[33] This suggests that it may be worthwhile to incorporate elements such as habit formation into the model. Since this is beyond the scope of the paper, it is left to future research.

A bigger trouble of the model, however, lies in predicting the volatility of hours. Hours in the US is less volatile than output whereas the predicted hours is about 40 percent more volatile than output, implying that predicted labor productivity is strongly countercyclical. This is inconsistent with empirical evidence. Such inconsistence is expected under demand shocks, however, due to diminishing marginal product of labor. Allowing for capacity utilization and labor hoarding may help mitigate this problem.[34]

6 Conclusion

The major contribution of the paper is three folded. 1) It documented stark differences in inventory behavior at different cyclical frequencies. 2) It provided a simple general-equilibrium explanation for the existence of inventories and the seemingly paradoxical features of inventory cycles. 3) It showed that consumption shocks have the potential to provide a unified explanation for both inventory fluctuations and the business cycle.

More work needs to be done, however, before one can claim that the inventory cycle and its

[33] As noted previously, with a higher persistence in preference shocks ($\rho = 0.99$), the model tends to generate a variance ration between production and sales that is greater than one at the high frequencies. This is inconsistent with the majority of OECD countries except for the US and Findland.

[34] Benhabib and Wen (2002) show that in a model of indeterminacy (resulting from variable capacity utilization and mild increasing returns to scale) demand shocks are capable of explaining procyclical labor productivity in spite of diminishing marginal product of labor.

relation to the business cycle are fully understood. For example, why does the general equilibrium model tend to over-predict inventory volatility in the short run (2-3 quarter cyclical frequencies) and under-predict inventory volatility in the longer run (at the business-cycle frequencies) as reflected in the $\varkappa$ index? Second, can the same general equilibrium theory explain the seasonal inventory cycle which seems to share a common pattern with the non-seasonal inventory cycle (e.g., see Blanchard 1983, Miron and Zeldes 1988)? Third, do the differences in inventory behavior at high and low cyclical frequencies also exist in disaggregate (firm-level or industry-level) data? Fourth, if the business cycle is indeed caused mainly by consumption demand shocks, then how to explain the countercyclical behavior of aggregate prices as documented by Kydland and Prescott (1990)? Inventories can certainly mitigate the positive impact of demand shocks on prices, but would that be sufficient to turn price movements completely countercyclical? Finally, How to interpret preference shocks? Do preference shocks reflect swings in people's mood (confidence), or do they reflect anticipated changes in other fundamentals such as future production possibilities? Or do they merely reflect sunspots? What this paper has shown is that consumption demand matters; but, what really causes consumption demand to change at the first place is debatable (another chicken-and-egg problem?). These important questions remain to be addressed in future research.

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Table 1. Stylized Facts from Different Countries

	High Freq. (2-3 Quarter)		B-C Freq. (8-40 Quarter)	
Countries	σ_y/σ_z	$cor(z,i)$	σ_y/σ_z	$cor(z,i)$
Australia	0.78	-0.62	1.29	0.04
Austria	0.49	-0.85	1.37	-0.02
Canada	0.80	-0.69	1.29	0.52
Denmark	0.79	-0.69	1.27	0.24
France	0.80	-0.68	1.57	0.14
Finland	1.22	-0.27	1.25	0.21
Great Britain	0.82	-0.57	1.33	0.35
Japan	0.81	-0.60	1.14	0.44
Netherlands	0.64	-0.79	1.37	0.37
Switzerland	0.65	-0.75	1.41	-0.06
United States	1.17	-0.24	1.20	0.60
OECD	0.91	-0.43	1.39	0.58
Europe (15)	0.81	-0.53	1.56	0.52
Europe	0.83	-0.51	1.55	0.47

*Data source: The US data are taken from the Citibase. The rest is taken from OECD data bank. All data are seasonally adjusted. The statistics reported do not change dramatically if the high-frequency interval is extended to 2-4 quarters per cycle and the business-cycle frequency interval is extended to 6-100 quarters per cycle.

Table 2. Predicted Sample Moments*

	High Freq. (2-3 Quarters)		B-C Freq. (8-40 Quarters)	
	σ_y/σ_c	$cor(c_t, \Delta s_t)$	σ_y/σ_c	$cor(c_t, \Delta s_t)$
Data**	0.91	-0.43	1.39	0.58
Model ($\phi = 0.1$)	0.64 (0.05)	-0.86 (0.01)	0.56 (0.02)	-0.81(0.02)
Model ($\phi = 1$)	0.95 (0.02)	-0.82 (0.01)	0.93 (.003)	-0.35 (0.02)
Model ($\phi = 10$)	1.51 (0.02)	-0.76 (0.01)	1.10(.001)	0.38(0.01)

*Numbers shown are the means of predicted moments based on 100 simulations (140 observations in each simulated sample). Standard errors are in parentheses.

**Data statistics refer to the OECD countries.

Table 3. Predicted Sample Moments*

	High Freq. (2-3 Quarters)		B-C Freq. (8-40 Quarters)	
	σ_y/σ_c	$cor(c_t, \Delta s_t)$	σ_y/σ_c	$cor(c_t, \Delta s_t)$
Data**	0.91	-0.43	1.39	0.58
Model ($\phi = 0.0$)	0.91(0.02)	-0.86 (0.01)	1.07 (0.001)	0.33 (0.01)
Model ($\phi = 1.0$)	0.95 (0.01)	-0.84 (0.01)	1.07 (0.001)	0.38 (0.01)
Model ($\phi = 5.0$)	1.17 (0.03)	-0.77 (0.02)	1.07 (0.001)	0.50 (0.01)

*Numbers shown are the means of predicted moments based on 100 simulations (140 observations in each simulated sample).

Standard errors are in parentheses.

**Data statistics are based on OECD aggregates.

Table 4. Predicted Sample Moments ($\phi = 0$)*

		High Frequencies		B-C Frequencies	
	$cor(c, \Delta k)$**	σ_y/σ_c	$cor(c_t, \Delta s_t)$	σ_y/σ_c	$cor(c_t, \Delta s_t)$
$\rho = 0.50$	-0.60 (.05)	0.13 (.000)	-0.999 (.000)	0.77 (.01)	-0.71 (.003)
$\rho = 0.97$	0.53 (.23)	0.91 (.02)	-0.86 (.01)	1.07 (.001)	0.33 (.01)
$\rho = 0.99$	0.85 (.08)	1.16 (.02)	-0.76 (.03)	1.06 (.001)	0.32 (.01)

*Numbers shown are the means of predicted moments based on 100 simulations (140 observations in each simulated sample).

Standard errors are in parentheses.

**This statistic is not decomposed by frequencies.

Table 5. Volatility Ratio of Inventory Investment to GDP across Frequencies

	r_H ($\omega \in [2/3\pi, \pi]$)	r_L ($\omega \in [0.05\pi, 0.25\pi]$)	$\varkappa\left(\frac{r_H}{r_L}\right)$
Australia	0.85	0.64	1.3
Austria	1.51	0.67	2.3
Canada	1.12	0.33	3.4
Denmark	1.37	0.45	3.0
France	1.24	0.69	1.8
Finland	0.77	0.47	1.6
Great Britain	0.89	0.41	2.2
Japan	0.55	0.21	2.6
Netherlands	1.01	0.49	2.1
Switzerland	1.34	0.82	1.6
United States	0.70	0.25	2.8
Sample Mean	1.03	0.49	2.2
Min	0.55	0.21	1.3
Max	1.51	0.82	3.4
Model ($\phi = 0$)	1.65 (0.12)	0.14 (0.001)	12 (1.8)
Model ($\phi = 1$)	1.54 (0.14)	0.13 (0.001)	12 (2.1)
Model ($\phi = 5$)	1.41 (0.09)	0.10 (0.001)	14 (2.8)

Table 6. Predicted Second Moments*

	σ_x/σ_y			$cor(x,y)$			$cor(x_t, x_{t-1})$			
	c	i	n	c	i	n	y	c	i	n
U.S. Data	0.79	3.18	0.82	0.87	0.89	0.84	0.84	0.86	0.89	0.82
M1	0.98	1.45	1.44	0.81	0.37	1.0	0.66	0.72	0.32	0.67
	(.06)	(.13)	(.01)	(.07)	(.24)	(.00)	(.08)	(.07)	(.18)	(.08)
M2**	0.68	2.57	1.25	0.85	0.85	0.99	0.65	0.74	0.70	0.65
	(.02)	(.18)	(.01)	(.05)	(.05)	(.00)	(.07)	(.05)	(.06)	(.07)

*Numbers shown in lower panel are the means of predicted moments based on 100 simulations

(140 observations in each simulated sample). Standard errors are in parentheses.

**Model 2 (M2) refers to $\alpha = 0.2, \rho = 0.99$.